MECHANICAL AND THERMAL STRESSES IN SUPERCONDUCTING ACCELERATOR AND BEAM LINE MAGNETS

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Summary

A solution has been found for stresses in a structural composite that models the superconducting magnets in the Energy Doubler and the High-Energy Beam Line. The composite consists of three nested hollow cylinders with the innermost cylinder representing the region of the bore tube, the middle cylinder the region of the superconductor, and the outermost cylinder the region of the collars. Under zero stress the distribution of current is chosen to give a pure dipole field. Subsequent effects of pre-stress, cooldown, and excitation on the state of stress are determined. The corresponding strains and effects of conductor movement on field quality are determined. Each region is characterized by two elastic constants, one thermal constant, and one pre-tension constant. Numerical results are given.

Introduction

A solution to a similar problem has been given previously[1] in which the magnetic field was generated by two sheet currents varying as cosine theta. One sheet current was located at the boundary between the innermost cylinder and the middle cylinder of structural material. The other sheet was located at the boundary between the middle cylinder and the outermost cylinder of structural material. The present note addresses the problem of improving the representation of the magnet excitation by replacing the two current sheets with a thick cosine theta current distribution in the middle structural region.

Equation for Elastic Displacement

If $\vec{u}$ is the displacemet vector and the body forces are derived from the Lorentz force then[2]

$$\nabla x \nabla x \vec{u} - 2\frac{1-\nu}{1-2\nu}\nabla(\nabla \cdot \vec{u}) = 2\frac{1+\nu}{E}\vec{J}x\vec{B} , \tag{1}$$

where E is Young's modules, ν is Poisson's ratio, $\vec{J}$ is the current density and $\vec{B}$ is the magnetic induction.

Generalized Plane Strain Approximation

For simplicity consider only the case for which $u_z = \varepsilon_{zz} z$ with ε_{zz} = constant. The remaining components are to be considered functions of (r, θ) only. This is consistent with an excitation in which J_z is the only component of current density. Hence Eq. (1) becomes

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial\theta}\right] - \beta\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right] = -2\,\frac{1+\nu}{E}J_z B_\theta , \tag{2}$$

$$-\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial\theta}\right] - \beta\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right] = 2\,\frac{1+\nu}{E}J_z B_r , \tag{3}$$

where

$$\beta = 2\frac{1-\nu}{1-2\nu} . \tag{4}$$

Calculation of Magnetic Quantities of Interest

By definition a thick cosine theta conductor carries an axial current between two radii (b, c) with a current density that varies as

$$J_z = J_o \cos\theta . \tag{5}$$

From this it follows[3] that

$$(r<b) \qquad B_r = -2\pi J_o(\lambda-b)\sin\theta, \qquad \text{(emu)} \tag{6}$$

$$B_\theta = -2\pi J_o(\lambda-b)\cos\theta, \qquad \text{(emu)} \tag{7}$$

$$(b<r<c) \quad B_r = \frac{2\pi}{3} J_o[2r-3\lambda+b^3r^{-2}]\sin\theta \tag{8}$$

$$B_\theta = \frac{2\pi}{3} J_o[4r-3\lambda-b^3r^{-2}]\cos\theta \tag{9}$$

$$(c<r<r_s) \quad B_r = -\frac{2\pi}{3} J_o(c^3-b^3)(r_s{}^{-2}+r^{-2})\sin\theta, \tag{10}$$

$$B_\theta = -\frac{2\pi}{3} J_o(c^3-b^3)(r_s{}^{-2}-r^{-2})\cos\theta, \tag{11}$$

where for convenience in these and subsequent formulas

$$\lambda = c+\frac{1}{3}(c^3-b^3)r_s{}^{-2}, \tag{12}$$

the radius of the iron shield being r_s. Hence in the region of the conductor

$$J_z B_\theta = \frac{\pi}{3} J_o{}^2[4r-3\lambda-b^3r^{-2}](1+\cos 2\theta), \qquad \text{(emu)} \tag{13}$$

$$J_z B_r = \frac{\pi}{3} J_o{}^2[2r-3\lambda+b^3r^{-2}]\sin 2\theta. \qquad \text{(emu)} \tag{14}$$

It is of interest to find expressions for the magnetic energy per unit length of the dipole and for the Maxwell stress tensor since this enters the virial theorem. The magnetic energy is given by

$$W_B = \frac{1}{8\pi} \iint (B_r{}^2+B_\theta{}^2) r dr d\theta \tag{15}$$

or utilizing Eqs. (6-11) one has

$$W_B = \frac{1}{3}\pi^2 J_o{}^2[\lambda(c^3-b^3)-\frac{1}{2}(c^4-b^4)-b^3(c-b)] . \tag{16}$$

The maximum current density J_o may be found in terms of the central field from Eq. (7)

$$B_o = -2\pi J_o(\lambda - b). \tag{17}$$

A cylindrical coordinate representation of the Maxwell stress tensor is

$$\overleftrightarrow{\tau} = \frac{1}{4\pi}\begin{pmatrix} B_r{}^2 - \frac{1}{2}B^2 & B_rB_\theta & B_rB_z \\ B_rB_\theta & B_\theta{}^2 - \frac{1}{2}B^2 & B_\theta B_z \\ B_rB_z & B_\theta B_z & B_z{}^2 - \frac{1}{2}B^2 \end{pmatrix}. \tag{18}$$

In utilizing this tensor in the virial theorem the only quantity needed is the projection of the outward radial traction on the radius vector. This becomes

$$\vec{r}\cdot\overleftrightarrow{\tau}\cdot\vec{n} = \frac{1}{8\pi}(B_r{}^2 - B_\theta{}^2)r\ . \tag{19}$$

<u>Form of Solution</u>

Equations (13) and (14) indicate that the form of the displacement should be taken as

$$u_r = P_o(r) + P_2(r)\cos 2\theta \qquad u_\theta = Q_2(r)\sin 2\theta\ . \tag{20}$$

Substituting into Eqs. (2-3) gives

$$-\beta\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rP_o)\right] = -\mu\left[4r - 3\lambda - b^3r^{-2}\right]. \tag{21}$$

$$\frac{4}{r^2}P_2 - \beta\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rP_2)\right] + \frac{2}{r^2}\frac{d}{dr}(rQ_2) - 2\beta\frac{d}{dr}\left(\frac{Q_2}{r}\right) = -\mu\left[4r - 3\lambda - b^3r^{-2}\right], \tag{22}$$

$$-2\frac{d}{dr}\left(\frac{P_2}{r}\right) + \frac{2\beta}{r^2}\frac{d}{dr}(rP_2) - \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rQ_2)\right] + \frac{4\beta}{r^2}Q_2 = \mu\left[2r - 3\lambda + b^3r^{-2}\right], \tag{23}$$

where for convenience

$$\mu = 2\frac{1+\nu}{E}\frac{\pi}{3}J_o^{\,2} . \tag{24}$$

Solutions of the Homogeneous Equation

In general these solutions are of the form

$$P_o = Ar^{-1}+Br \qquad P_2 = Cr^p \qquad Q_2 = Dr^p \tag{25}$$

where p is found by substituting into Eqs. (22-23) to obtain

$$[4-\beta(p^2-1)]C+2[p+1-\beta(p-1)]D = 0 , \tag{26}$$

$$2[-p+1+\beta(p+1)]C-[p^2-1-4\beta]D = 0 . \tag{27}$$

The determinant of the coefficients is

$$\Delta(p) = \beta(p^2-1)(p^2-9) . \tag{28}$$

Setting this equal to zero gives $p = \pm 1, \pm 3$. Hence there are four solutions which must be added together to give

$$P_2 = -D_1r-\beta D_2r^{-1}-\frac{2-\beta}{1-2\beta}D_3r^3+D_4r^{-3}, \tag{29}$$

$$Q_2 = D_1r+D_2r^{-1}+D_3r^3+D_4r^{-3}. \tag{30}$$

Thus the homogeneous solutions are seen to be identical in form with those found previously[1] after it is recognized that ($u_r = \frac{1}{\beta}Gr\ln r$, $u_\theta = Gr\theta$) the solution describing pretension may also be added.

Particular Solution

Since each term of the RHS of Eq. (21) is of the form Br^q, one may take for P_o in Eq. (20): $P_o = Ar^p$. Then $p = q+2$ and

$$A = -\frac{1}{\beta}\frac{B}{p^2-1} . \tag{31}$$

Adding the contributions for $q = 1, 0, -2$ gives

$$P_o = \frac{\mu}{\beta}\left[\frac{1}{2}r^2 - \lambda r^2 + b^3\right]. \tag{32}$$

For the remainder of the solution one may take $\begin{pmatrix} P_2 \\ Q_2 \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} r^p$ corresponding to $\begin{pmatrix} E \\ F \end{pmatrix} r^q$ as a term on the RHS of Eqs. (22-23). Substituting into Eqs. (22-23) gives

$$\begin{pmatrix} 4-\beta(p^2-1) & 2[p+1-\beta(p-1)] \\ 2[-p+1+\beta(p+1)] & -p^2+1+4\beta \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} r^{p-2} = \begin{pmatrix} E \\ F \end{pmatrix} r^q. \tag{33}$$

Equation (28) gives the determinant of the coefficients. Hence, the inversion of Eq. (33) gives $p = q+2$ and

$$\begin{pmatrix} C \\ D \end{pmatrix} = \frac{1}{\Delta(p)}\begin{pmatrix} -p^2+1+4\beta & -2[p+1-\beta(p-1)] \\ -2[-p+1+\beta(p+1)] & 4-\beta(p^2-1) \end{pmatrix}\begin{pmatrix} E \\ F \end{pmatrix}. \tag{34}$$

Note that the q-values of interest are $q = 1, 0, -2$ or $p = 3, 2, 0$. Equation (34) can provide solutions only for $p = 2, 0$ since $p = 3$ gives $\Delta(3) = 0$. In this case one considers

$$P_2 = (A+B\ln r)r^3 \qquad Q_2 = (C+D\ln r)r^3 . \tag{35}$$

Substituting these forms into Eqs. (22-23) for the $q = 1$ component of the RHS yields

$$\begin{aligned} &[(4-8\beta)A+(8-4\beta)C-6\beta B+2(1-\beta)D]r \\ &+[(4-8\beta)B+(8-4\beta)D]r\ln r = -4\mu r, \end{aligned} \tag{36}$$

$$\begin{aligned} &-[(4-8\beta)A+(8-4\beta)C+2(1-\beta)B+6D]r \\ &-[(4-8\beta)B+(8-4\beta)D]r\ln r = 2\mu r. \end{aligned} \tag{37}$$

To eliminate the $r\ln r$ term in both Eq. (36) and (37) choose

$$(1-2\beta)B+(2-\beta)D = 0. \tag{38}$$

Equations (36-37) then become

$$(4-8\beta)A+(8-4\beta)C-6\beta B+2(1-\beta)D = -4\mu, \tag{39}$$

$$(4-8\beta)A+(8-4\beta)C+2(1-\beta)B+6D = -2\mu. \tag{40}$$

Subtracting Eq. (39) from Eq. (40) gives

$$(1+2\beta)B+(2+\beta)D = \mu. \tag{41}$$

Solving Eqs. (38) and (41) simultaneously

$$B = \frac{1}{6}\,\frac{2-\beta}{\beta}\,\mu \qquad D = -\frac{1}{6}\,\frac{1-2\beta}{\beta}\,\mu. \tag{42}$$

Equation (39) or (40) now becomes

$$(1-2\beta)A+(2-\beta)C = \frac{1}{12}\,\frac{1-9\beta-\beta^2}{\beta}\,\mu. \tag{43}$$

Since (A, C) are coefficients of r^3 terms and already included in the homogeneous solution one may choose one relation between A and C arbitrarily. Let $C = -2A$ in order to remove the β-terms from the LHS of Eq. (43). Then

$$A = -\frac{1}{36}\,\frac{1-9\beta-\beta^2}{\beta}\,\mu \qquad C = \frac{1}{18}\,\frac{1-9\beta-\beta^2}{\beta}\,\mu. \tag{44}$$

Displacement

The particular solution may be thought of as an incremental displacement to be added to the homogeneous terms. Thus assembling all the parts from Eq. (31) to Eq. (44) one has

$$\Delta u_r = \frac{\mu}{\beta}\Big\{\frac{1}{2}r^3-\lambda r^2+b^3+[-\frac{1}{9}(1-2\beta)b^3-\frac{1}{5}(3+2\beta)\lambda r^2$$
$$-\frac{1}{36}(1-9\beta-\beta^2)r^3+\frac{1}{6}(2-\beta)r^3\ln r]\cos 2\theta\Big\}, \tag{45}$$

$$\Delta u_\theta = \frac{\mu}{\beta}\Big\{\frac{1}{9}(2-\beta)b^3+\frac{1}{5}(2+3\beta)\lambda r^2$$
$$+\frac{1}{18}(1-9\beta-\beta^2)r^3-\frac{1}{6}(1-2\beta)r^3\ln r\Big\}\sin 2\theta. \tag{46}$$

Strain

The incremental strain to be added to the homogeneous solution is found from Eqs. (45-46) using Eqs. (61-63) of Ref. (1).

$$\Delta\varepsilon_{rr} = \frac{\mu}{\beta}\left\{\frac{3}{2}r^2-2\lambda r+\left[-\frac{2}{5}(3+2\beta)\lambda r +\frac{1}{12}(3+7\beta+\beta^2)r^2+\frac{1}{2}(2-\beta)r^2\ln r\right]\cos 2\theta\right\}, \tag{47}$$

$$\Delta\varepsilon_{\theta\theta} = \frac{\mu}{\beta}\left\{\frac{1}{2}r^2-\lambda r+b^3r^{-1}+\left[\frac{1}{3}b^3r^{-1}+\frac{1}{5}(1+4\beta)\lambda r +\frac{1}{12}(1-9\beta-\beta^2)r^2+\frac{1}{2}\beta r^2\ln r\right]\cos 2\theta\right\}, \tag{48}$$

$$\Delta\varepsilon_{r\theta} = \frac{\mu}{\beta}\left\{-\frac{1}{6}\beta b^3r^{-1}+\frac{1}{10}(8+7\beta)\lambda r -\frac{1}{12}\beta(7+\beta)r^2-\frac{1}{2}(1-\beta)r^2\ln r\right\}\sin 2\theta. \tag{49}$$

Stress

The incremental stress to be added to the homogeneous solution is found from Eqs. (47-49) using Eqs. (48-50) of Ref. (1). After inverting one first has

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}\left[\frac{1}{2}\beta\Delta\varepsilon_{rr}-\frac{1}{2}(2-\beta)\Delta\varepsilon_{\theta\theta}\right], \tag{50}$$

$$\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}\left[-\frac{1}{2}(2-\beta)\Delta\varepsilon_{rr}+\frac{1}{2}\beta\Delta\varepsilon_{\theta\theta}\right], \tag{51}$$

$$\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\,\varepsilon_{r\theta}. \tag{52}$$

Then using Eqs. (47-49) for the incremental strain one finds

$$\Delta\sigma_{rr} = \frac{2\pi}{3\beta}J_o^{\,2}\left\{-\frac{1}{2}(1-2\beta)r^2+\frac{1}{2}(2-3\beta)\lambda r-\frac{1}{2}(2-\beta)b^3r^{-1} + \left[-\frac{1}{6}(2-\beta)b^3r^{-1}-\frac{1}{10}(2+13\beta)\lambda r -\frac{1}{12}(1-11\beta)r^2\right]\cos 2\theta\right\}, \tag{53}$$

$$\Delta\sigma_{\theta\theta} = \frac{2\pi}{3\beta}J_o^{\,2}\Bigg\{-\tfrac{1}{2}(3-2\beta)r^2+\tfrac{1}{2}(4-3\beta)\lambda r+\tfrac{1}{2}\beta b^3 r^{-1}$$

$$+\Bigg[\tfrac{1}{6}b^3 r^{-1}+\tfrac{3}{10}(4+\beta)\lambda r$$

$$-\tfrac{1}{12}(3+5\beta+2\beta^2)r^2-(1-\beta)r^2\ln r\Bigg]\cos 2\theta\Bigg\}, \tag{54}$$

$$\Delta\sigma_{r\theta} = \frac{2\pi}{3\beta}J_o^{\,2}\Bigg\{-\tfrac{1}{6}\beta b^3 r^{-1}+\tfrac{1}{10}(8+7\beta)\lambda r$$

$$-\tfrac{1}{12}\beta(7+\beta)r^2-\tfrac{1}{2}(1-\beta)r^2\ln r\Bigg\}\sin 2\theta. \tag{55}$$

Boundary Conditions

Since the form of the solution of the homogeneous equation is identical with that of Ref. (1) one may utilize Eqs. (55-65) of that reference. To these equations add the particular solutions found here to give the correct expressions for displacement, strain and stress. Apart from the special condition related to pretension and discussed in Ref. (1) the boundary conditions are as follows:

At r=a, the innermost radius $\quad \sigma_{rr}^{(+)} = \sigma_{r\theta}^{(+)} = 0$

At r=b $\quad \sigma_{rr}^{(+)}-\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)}-\sigma_{r\theta}^{(-)} = 0 \quad (57)$

$$u_r^{(+)}-u_r^{(-)} = u_\theta^{(+)}-u_\theta^{(-)} = 0 \tag{58}$$

At r=c $\quad \sigma_{rr}^{(+)}-\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)}-\sigma_{r\theta}^{(-)} = 0 \quad (59)$

$$u_r^{(+)}-u_r^{(-)} = u_\theta^{(+)}-u_\theta^{(-)} = 0 \tag{63}$$

At r=d, the outermost radius $\quad \sigma_{rr}^{(-)} = \sigma_{r\theta}^{(-)} = 0. \quad (61)$

Use of the Virial Theorem

As discussed in Ref. (1) the virial theorem is used to obtain enough conditions to permit the longitudinal strain to be determined. From Eq. (95) in Ref. (1) the virial theorem may be written as

$$\iint(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})rdrd\theta = \int_{r=r_s}\vec{r}\cdot\vec{\vec{\tau}}\cdot\vec{n}rd\theta+W_B, \tag{62}$$

where the double integral is throughout the cross section of the material under stress. The single integral is over the cylinder at $r = r_s$ and W_B is the magnetic energy per unit length contained within the region bounded by $r = r_s$. From Eqs. (16) and (19) one has

$$\iint(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})rdrd\theta = \frac{1}{3}\pi^2 J_o^{\,2}\left\{-\frac{1}{2}(c^4-b^4)\right.$$

$$\left.+[c+(c^3-b^3)r_s^{\,-2}](c^3-b^3)-b^3(c-b)\right\}. \qquad \text{(dynes)} \tag{63}$$

Since the stress components are the sum of the homogeneous and particular solutions and since the present homogeneous solutions are identical in form with those of Ref. (1), the LHS of Eq. (102) in Ref. (1) must be enhanced by $\iint(1+\nu)(\Delta\sigma_{rr}+\Delta\sigma_{\theta\theta})rdrd\theta$. In addition the current dependent terms on the RHS of Eq. (102) in Ref. (1) must be replaced with the RHS of Eq. (63). Using Eqs. (53-54) one finds

$$\iint(1+\nu)(\Delta\sigma_{rr}+\Delta\sigma_{\theta\theta})rdrd\theta = \frac{2}{3}\pi^2 J_o^{\,2}\,\frac{1-\beta}{\beta}(1+\nu)\ \cdot$$

$$[-(c^4-b^4)+2\lambda(c^3-b^3)-2b^3(c-b)]\quad . \tag{64}$$

Numerical Results

The stresses and strains that exist in three nested hollow cylinders have been calculated as a function of the central magnetic field. This has been done by utilizing the basic book-keeping for the homogeneous part of the solution from a previous[1] calculation. The improvement here consists of replacing the sheet current excitation of the previous problem with a thick cosine theta excitation in the region of the middle cylinder. As before twenty algebraic relations stemming from the boundary conditions in Eqs. (56-61), the pretension condition, and the virial theorem have been utilized to determine twenty coefficients. Thus the stress, strain, and displacement of any point in the dipole model structure may be found. For simplicity in the presentation of numerical results, however, only the values of $r = a$, $r = b$, $r = c$ and $r = d$ are given for a few angles. It is usually clear whether a quantity is stress or strain. Otherwise, R is radial, T is theta or azimuthal, Z is axial or longitudinal. To indicate the side of a point, P is used for positive and M for negative. Thus, for example, RTBP indicates the (r,θ) component at the positive side of the point $r = b$. The quantity $\sqrt{3J_2}$ which is to be compared with the yield stress is tension is explained in Ref. (1).

References

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ELASTIC STRESS AND STRAIN IN FOUR SHELL BEAM LINE DIPOLE

CENTRAL MAGNETIC FIELD(KG) = 0.0000 INNER BORE TUBE RADIUS(IN) = 3.0000 INNER CONDUCTOR RADIUS(IN)) = 3.5000
OUTER CONDUCTOR RADIUS(IN)) = 5.0000 OUTER RADIUS OF BANDING(IN) = 6.0000 RADIUS OF IRON SHIELD(IN) = 6.0000
BORE YOUNGS MODULUS(LBS/IN/IN) = 10000000. COND. YOUNGS MODULUS(LBS/IN/IN)= 1000000. BAND YOUNGS MODULUS(LBS/IN/IN) = 10000000.
BORE POISSONS RATIO = .3330 COND. POISSONS RATIO = .1000 BAND POISSONS RATIO = .3330
BORE THERMAL STRAIN(IN/IN) = -.0020 COND. THERMAL STRAIN(IN/IN) = -.0030 BAND THERMAL STRAIN(IN/IN) = -.0040
BAND DISLOCATION(MILLIRAD) = 5.250 MAGNETIC ENERGY((IN-LBS)/IN) = 0.00 MAGNETIC ENERGY(J/M) = 0.00

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
80.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
70.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
60.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
50.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
40.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
30.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
20.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
10.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.
.0	0.	-22601.	0.	-2998.	-19603.	0.	-2998.	-480.	0.	-2356.	-1122.	0.	-2356.	12155.	0.	0.	11513.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
80.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
70.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
60.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
50.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
40.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
30.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
20.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
10.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.
.0	-475.	-3488.	0.	-875.	-3088.	0.	-5859.	-3088.	0.	-5152.	-3795.	0.	-4894.	-2959.	0.	-4656.	-3121.	0.

ANGLE (DEG)	LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	SQRT(3*J2) IN (KLB/IN/IN) YAP	YBM	YBP	YCM	YCP	YDM	TRANSVERSE DISPLACEMENT(IN) URA	UTA	URB	UTB	URC	UTC	URD	UTD
90.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0079
80.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0070
70.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0061
60.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0053
50.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0044
40.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0035
30.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0026
20.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0018
10.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0009
.0	-23183.	-23183.	-913.	-913.	7606.	8176.	-3566.	22.9	18.7	2.3	1.4	12.9	10.3	-.01050	.0000	-.01080	.0000	-.01900	.0000	-.0246	.0000

ELASTIC STRESS AND STRAIN IN FOUR SHELL BEAM LINE DIPOLE

CENTRAL MAGNETIC FIELD(KG)	= 43.5000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	=	3.5000	
OUTER CONDUCTOR RADIUS(IN))	= 5.0000	OUTER RADIUS OF BANDING(IN)	= 6.0000	RADIUS OF IRON SHIELD(IN)	=	8.0000	
BORE YOUNGS MODULUS(LBS/IN/IN)	= 10000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	=	10000000.	
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	=	.3330	
BORE THERMAL STRAIN(IN/IN)	= -.0020	COND. THERMAL STRAIN(IN/IN)	= -.0030	BAND THERMAL STRAIN(IN/IN)	=	-.0040	
BAND DISLOCATION(MILLIRAD)	= 5.250	MAGNETIC ENERGY((IN-LBS)/IN)	= 89479.62	MAGNETIC ENERGY(J/M)	=	394136.50	

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-0.	-2651.	-0.	-1772.	-14183.	0.	-1772.	53.	0.	-2122.	-699.	0.	-2122.	17085.	0.	0.	6774.	0.
80.0	-0.	-3722.	0.	-1828.	-14395.	-301.	-1828.	30.	-901.	-2145.	-723.	-160.	-2145.	16838.	-160.	0.	7100.	-0.
70.0	-0.	-6806.	0.	-1991.	-15005.	-1693.	-1991.	-35.	-1693.	-2211.	-791.	-300.	-2211.	16125.	-300.	0.	8041.	-0.
60.0	-0.	-11530.	0.	-2240.	-15943.	-2281.	-2240.	-136.	-2281.	-2313.	-896.	-404.	-2313.	15033.	-404.	0.	9482.	-0.
50.0	-0.	-17326.	0.	-2545.	-17091.	-2594.	-2545.	-259.	-2594.	-2438.	-1025.	-459.	-2438.	13694.	-459.	0.	11249.	-0.
40.0	0.	-23493.	0.	-2870.	-18313.	-2594.	-2870.	-391.	-2594.	-2571.	-1161.	-459.	-2571.	12259.	-459.	0.	13137.	-0.
30.0	0.	-29289.	0.	-3175.	-19462.	-2281.	-3175.	-514.	-2281.	-2696.	-1290.	-404.	-2696.	10929.	-404.	0.	14897.	-0.
20.0	0.	-34013.	0.	-3424.	-20398.	-1693.	-3424.	-615.	-1693.	-2798.	-1395.	-300.	-2798.	9837.	-300.	0.	16338.	-0.
10.0	0.	-37097.	0.	-3587.	-21009.	-901.	-3587.	-680.	-901.	-2864.	-1463.	-160.	-2864.	9124.	-160.	0.	17279.	-0.
.0	0.	-38168.	0.	-3643.	-21222.	-0.	-3643.	-703.	-0.	-2887.	-1487.	-0.	-2887.	8876.	-0.	0.	17605.	-0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-1411.	-1765.	-0.	-1057.	-2711.	0.	-4719.	-2711.	0.	-4982.	-3417.	0.	-5142.	-2582.	0.	-4495.	-3593.	0.
80.0	-1354.	-1860.	0.	-1052.	-2728.	-120.	-4772.	-2728.	-991.	-5003.	-3438.	-175.	-5133.	-2603.	-21.	-4510.	-3563.	-0.
70.0	-1227.	-2134.	0.	-1040.	-2775.	-226.	-4925.	-2775.	-1862.	-5061.	-3499.	-330.	-5107.	-2663.	-40.	-4552.	-3480.	-0.
60.0	-1017.	-2554.	0.	-1020.	-2347.	-304.	-5161.	-2847.	-2519.	-5150.	-3591.	-444.	-5068.	-2756.	-54.	-4616.	-3352.	-0.
50.0	-750.	-3069.	0.	-996.	-2935.	-345.	-5449.	-2935.	-2854.	-5260.	-3705.	-505.	-5019.	-2869.	-61.	-4694.	-3195.	-0.
40.0	-496.	-3618.	0.	-971.	-3030.	-345.	-5757.	-3030.	-2854.	-5376.	-3825.	-505.	-4968.	-2990.	-61.	-4778.	-3027.	-0.
30.0	-229.	-4133.	0.	-947.	-3118.	-304.	-6045.	-3118.	-2509.	-5486.	-3939.	-444.	-4920.	-3103.	-54.	-4855.	-2870.	-0.
20.0	-19.	-4553.	0.	-928.	-3190.	-226.	-6281.	-3190.	-1863.	-5575.	-4032.	-330.	-4880.	-3195.	-40.	-4920.	-2742.	-0.
10.0	118.	-4827.	0.	-915.	-3238.	-120.	-6434.	-3238.	-991.	-5633.	-4092.	-176.	-4854.	-3256.	-21.	-4962.	-2659.	-0.
.0	165.	-4922.	0.	-911.	-3254.	-0.	-6488.	-3254.	-0.	-5653.	-4113.	-0.	-4845.	-3277.	-0.	-4976.	-2629.	-0.

ANGLE (DEG)	LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	SQRT(3*J2) IN (KLB/IN/IN) YAP	YBM	YBP	YCM	YCP	YDM	TRANSVERSE DISPLACEMENT(IN) URA	UTA	URB	UTB	URC	UTC	URD	UTD
90.0	-15033.	-19463.	-587.	-697.	10932.	8105.	-3415.	13.9	15.7	1.6	1.4	17.0	7.5	-.0196	.0000	-.0202	.0000	-.0277	.0000	-.0333	.0079
80.0	-15390.	-19553.	-595.	-702.	10742.	8214.	-3415.	13.9	15.9	2.3	1.5	16.8	7.7	-.0191	-.0025	-.0197	-.0018	-.0272	-.0018	-.0328	.0060
70.0	-16417.	-19810.	-618.	-715.	10493.	8527.	-3415.	14.3	16.2	3.4	1.5	16.3	8.3	-.0174	-.0046	-.0190	-.0035	-.0256	-.0034	-.0313	.0042
60.0	-17990.	-20205.	-653.	-736.	10085.	9007.	-3415.	15.8	16.7	4.4	1.7	15.5	9.3	-.0149	-.0062	-.0153	-.0047	-.0233	-.0046	-.0289	.0027
50.0	-19920.	-20685.	-695.	-761.	9598.	9596.	-3415.	18.8	17.2	5.0	1.8	14.5	10.5	-.0117	-.0071	-.0121	-.0053	-.0204	-.0052	-.0260	.0014
40.0	-21974.	-21204.	-741.	-788.	9079.	10222.	-3415.	22.8	17.7	5.1	1.8	13.6	11.9	-.0084	-.0071	-.0087	-.0053	-.0173	-.0052	-.0230	.0006
30.0	-23904.	-21689.	-784.	-814.	8591.	10810.	-3415.	27.0	17.9	4.7	1.8	12.6	13.3	-.0052	-.0062	-.0055	-.0047	-.0144	-.0046	-.0201	.0001
20.0	-25477.	-22083.	-819.	-834.	8194.	11290.	-3415.	30.7	18.1	4.0	1.8	11.9	14.5	-.0027	-.0046	-.0029	-.0035	-.0120	-.0034	-.0177	-.0002
10.0	-26504.	-22341.	-842.	-848.	7934.	11603.	-3415.	33.1	18.2	3.2	1.8	11.4	15.3	-.0010	-.0025	-.0012	-.0018	-.0105	-.0018	-.0162	-.0001
.0	-26860.	-22430.	-850.	-852.	7844.	11712.	-3415.	34.0	18.2	2.9	1.8	11.3	15.5	-.0004	-.0000	-.0006	-.0000	-.0099	-.0000	-.0157	-.0000